WATER RESOURCES RESEARCH, VOL. 32, NO. 5, PAGES 1251-1258, MAY 1996

Parameter equivalence for the Brooks-Corey and van Genuchten soil characteristics: Preserving the effective capillary drive

Hubert J. Morel-Seytoux,[1] Philip D. Meyer,[2] Mahmood Nachabe,[3] Jaoudat Touma,[4] M. T. van Genuchten,[5] and Robert J. Lenhard[2]

Abstract. This paper provides a simple way to convert Brooks-Corey (BC) parameters to van Genuchten (vG) parameters and vice versa, for use primarily in situations where saturated conditions are likely to be encountered. Essential in this conversion is the preservation of the maximum value of a physical characteristic, the "effective capillary drive" H_{cM} [*Morel-Seytoux and Khanji*, 1974], defined with a good approximation for a soil water and air system as $H_{cM} = \int_0^\infty k_{rw}\, dh_c$, where k_{rw} is relative permeability (or conductivity) to water and h_c is capillary pressure (head), a positive quantity. With this conversion, infiltration calculations are essentially insensitive to the model used to represent the soil hydraulic properties. It is strictly a matter of convenience for the user which expression is used. On the other hand, the paper shows that other equivalences may lead to great variations in predictions of infiltration capacity. Consequently, the choice of the proper equivalence to use in calculations for rainfall-runoff modeling or for low-level radioactive waste disposal design is a serious matter.

Introduction

Depending on circumstances, it is convenient for analytical derivations or for numerical work to favor one type of expression for the hydraulic conductivity and water retention properties over another. However, the question can be legitimately raised as to the influence of that choice on the results. To answer this question conclusively, one would need to carry out the computations twice, once with each expression. However, it is sometimes not possible to perform the analytical derivations for both. For analytical derivations the *Brooks and Corey* [1964] (BC) formulae tend to be easier to use. On the other hand, some codes for numerical solution of the unsaturated flow equation use the *van Genuchten* [1980] (vG) expressions exclusively. If codes provide both options, it still remains to define the values of the parameters for both sets of expressions. Even if the data, on which one set of expressions was calibrated, and the criterion used for calibration were readily available, the criterion for goodness of fit of the other expression is, as always in calibration, somewhat arbitrary and furthermore need not be the same. As in statistical analysis, when different probability density functions with a small number of parameters (say two) are fitted to the same data, one must decide which density characteristic is to be preserved. That same decision must be made to define "equivalent" parameters for the Brooks-Corey and van Genuchten expressions. In what sense are they to be equivalent? Curve fitting different analytical expressions to the same set of data, by the classical unconstrained least squares technique, will not inherently preserve an "important" characteristic contained but somewhat hidden in the data. This naturally begs the question: What is an important characteristic to be preserved?

Surprisingly, few formal attempts [e.g., *Lenhard et al.*, 1989; *Russo et al.*, 1991] at defining an equivalence have been reported in the literature, because, clearly, it is an issue that must have confronted many code users and modelers.

Objective

The main objective of this paper is to provide a simple way to convert Brooks-Corey (BC) parameters to van Genuchten (vG) parameters and vice versa for use primarily in situations where saturated conditions are likely to be encountered. There are applications where the proper description of the soil properties at high water contents is crucial. Watershed rainfall-runoff modeling is one, and capillary barrier design is another. Essential in this conversion is the preservation of the maximum value of a well-defined physical characteristic, the effective capillary drive H_{cM} [*Morel-Seytoux and Khanji*, 1974, 1975], given with a good approximation for a soil water and air system as

$$H_{cM} = \int_0^\infty k_{rw}\, dh_c \tag{1}$$

where k_{rw} is relative permeability (or conductivity) to water and h_c is capillary pressure (head), or suction, a positive quantity, the negative of the matric head. H_{cM} has received other names in the literature, such as "macroscopic capillary length" [*Philip*, 1985; *White and Sully*, 1987].

[1]Department of Civil Engineering, Colorado State University, Fort Collins.
[2]Pacific Northwest Laboratories, Richland, Washington.
[3]Department of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder.
[4]Departement des Eaux Continentales, Institut Français de Recherche Scientifique pour le Développement en Coopération (ORSTOM), Montpellier, France.
[5]U.S. Salinity Laboratory, USDA Agricultural Research Service, Riverside, California.

Paper number 96WR00069.
0043-1397/96/96WR-00069$09.00

Physical Significance of H_{cM}

There are an infinite number of ways to define an equivalence between the two sets of parameters. Thus there must be a physical justification for the one proposed here. It is known [e.g., *Morel-Seytoux and Khanji*, 1974] that the effective capillary drive plays a significant role in determining the infiltration rate into a vertical soil column under a ponded condition at the surface. Under such conditions one can show [*Morel-Seytoux and Khanji*, 1974] that the "infiltration capacity" (which is the infiltration rate when infiltration takes place at capacity), thus more precisely defined as the capacity infiltration rate, is given by the expression

$$i = \bar{K}[(\bar{\theta} - \theta_i)H_{cM} + I]/\beta I \qquad (2)$$

where $\bar{K}$ is hydraulic conductivity at natural saturation (or field saturation, that is, porosity minus trapped air content), $\bar{\theta}$ is (volumetric) water content at natural saturation, θ_i is uniform initial water content, I is cumulative infiltration depth up to time t, and β, the viscous correction factor, is a dimensionless parameter which theory [*Morel-Seytoux*, 1969] and numerical experiments [*Morel-Seytoux and Billica*, 1985] indicate is essentially a constant for a given soil and varies little from soil to soil. From sand to clay, *Morel-Seytoux and Khanji* [1974] found a range in β of 1.1 to 1.7 and estimated an average value of 1.3. In the case $\beta = 1$, (2) has precisely the functional form of the Green-Ampt formula [e.g., *Morel-Seytoux and Khanji*, 1974]. The parameter β deviates from 1 for two reasons. First, the real wetting front profile does not have a saturated rectangular shape. Second, in the wetted region, water and air flow simultaneously, and the overall viscous resistance of the complex mixture exceeds that of water alone.

From (2) an expression for the two-phase sorptivity [*Morel-Seytoux and Khanji*, 1974, 1975] can be defined as the constant S_2 in the expression for infiltration rate at early times, namely,

$$i = \frac{1}{2} S_2 \frac{1}{\sqrt{t}} = \frac{1}{2}\sqrt{\frac{2(\bar{\theta} - \theta_i)H_{cM}\bar{K}}{\beta}}\frac{1}{\sqrt{t}} \qquad (3)$$

Equation (3) defines S_2 in terms of the primary parameters H_{cM} and $\bar{K}$. It has also been shown [e.g., *Warrick and Broadridge*, 1992], but from the single-phase flow point of view, that

$$S_1 = \sqrt{\frac{2(\bar{\theta} - \theta_i)H_{cM}\bar{K}}{2b}} \qquad (4)$$

The only difference between S_1 and S_2 is in the estimation of β and $2b$. *White and Sully* [1987] recommended for b a value of 0.55, regardless of soil, thus $2b = 1.10$. Given the fact that $\beta = 1.30$ includes the viscous resistance of the air phase (neglected in Richards equation) in addition to the deviation of the water content profile from a rectangular shape, one would expect β to be somewhat larger than $2b$. The numbers are remarkably close. For a value of $2b = 1.1$ the early infiltration rate is 95% that given by the Green-Ampt formula, and with $\beta = 1.3$ the early infiltration rate is 88% of it. Given the relative constancy of β or $2b$, the value of H_{cM} can be obtained with good accuracy by an infiltration test. The steady state value reached by the infiltration test yields $\bar{K}$. The early part of the infiltration test yields H_{cM}. (Naturally, the entire observed curve can be used to perform the calibration, for example, by using a least squares fit.) The value of $\bar{\theta}$ is measured at the end of the experiment; that of θ_i is measured at the beginning of the experiment.

Brooks-Corey and van Genuchten Expressions

Soil Hydrologic Characteristics

Normalized water content is defined as

$$\theta^* = (\theta - \theta_r)/(\bar{\theta} - \theta_r) \qquad (5)$$

where θ is (volumetric) water content and θ_r is residual water content. The BC expressions are

$$h_c = h_{ce}\theta^{*-M} \qquad h_c \geq h_{ce} \qquad (6)$$

$$k_{rw} = \theta^{*p} \qquad h_c \geq h_{ce} \qquad (7a)$$

$$k_{rw} = 1 \qquad h_c \leq h_{ce} \qquad (7b)$$

No physical significance is hereby attached to the parameter h_{ce} (also denoted h_{cb}) even though it is called herein entry (or bubble) pressure. In the application for infiltration problems these must be the curves corresponding to wetting conditions. To reduce the number of parameters to two, a relation developed by *Corey* [1977] is used:

$$p = 3 + 2M \qquad (8a)$$

or

$$M = (p - 3)/2 \qquad (8b)$$

The vG expressions are

$$h_c = \frac{1}{\alpha}[\theta^{*-(1/m)} - 1]^{1/n} \qquad (9)$$

where α^{-1} is a parameter with dimension of length. In the case when the following relation between m and n holds:

$$m = 1 - (1/n) \qquad (10a)$$

or

$$n = 1/(1 - m) \qquad (10b)$$

then

$$k_{rw} = \theta^{*(1/2)}\{1 - [1 - \theta^{*(1/m)}]^m\}^2 \qquad (11)$$

To reduce the vG expressions to two parameters, (10) is always implied throughout this article.

Applicability of the BC Expressions Near or at Saturation

It is sometime stated that the BC function does not apply to the saturated portion of $\theta(h_c)$. Whereas it is correct that (6) does not apply for $h_c \leq h_{ce}$, k_{rw} is nevertheless a continuous function of h_c, keeping the value 1 in the capillary fringe ($0 \leq h_c \leq h_{ce}$). At $h_c = h_{ce}$ there is a discontinuity in slope of $k_{rw}(h_c)$, but the integral given by (1) is perfectly regular and integrable at that point. Note that the variable of integration is h_c not θ or θ^*. Specifically, consider the integral defined by (1) and carry out the integration step by step as indicated presently:

$$H_{cM} = \int_0^\infty k_{rw}\, dh_c = \int_0^{h_{ce}} k_{rw}\, dh_c + \int_{h_{ce}}^\infty k_{rw}\, dh_c$$

$$H_{cM} = \int_0^{h_{ce}} dh_c + \int_1^0 k_{rw}\frac{dh_c}{d\theta^*}\, d\theta^* = h_{ce}$$

$$+\int_0^1 k_{rw}\left(-\frac{dh_c}{d\theta^*}\right) d\theta^* \quad (12)$$

$$H_{cM} = h_{ce} + h_{ce}\int_0^1 k_{rw} M\theta^{*-(M+1)}\, d\theta^* = h_{ce}$$

$$+ Mh_{ce}\int_0^1 \theta^{*p}\theta^{*-(M+1)}\, d\theta^*$$

$$H_{cM} = h_{ce} + h_{ce}\frac{M}{p-M}\theta^{*p-M}\Big|_0^1 = h_{ce} + \frac{M}{p-M}h_{ce}$$

$$= \frac{p}{p-M}h_{ce}$$

This derivation demonstrates the need in numerical work to deal with a pressure-based formulation or better yet with a mixed pressure-water content formulation as demonstrated previously [*Morel-Seytoux and Billica*, 1985].

Effective Capillary Drive

In the case of the BC expressions the integral in (1) can be expressed as

$$H_{cM} = h_{ce}p/(p - M) \quad (13)$$

which, when combined with (8), yields

$$H_{cM} = h_{ce}2p/(p + 3) \quad (14a)$$

or

$$h_{ce} = H_{cM}(p + 3)/2p \quad (14b)$$

If the physical quantity of interest to be preserved is H_{cM}, the calibration of the parameters h_{ce} and p must be subject to the constraint imposed by (14). In the case of the vG expressions, the integral of definition for H_{cM} is complex. However, it can be done numerically (see appendix) and can be approximated accurately by the expression,

$$H_{cM} = \left(\frac{1}{\alpha}\right)\left(\frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2}\right) \quad (15)$$

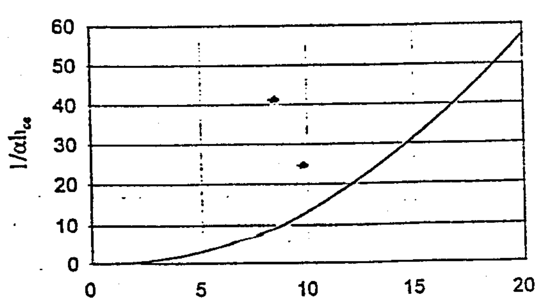

Figure 1. Dimensionless ratio $(1/\alpha)/h_{ce}$ as a function of the parameter p, exponent of normalized water content for the power law expression of relative permeability.

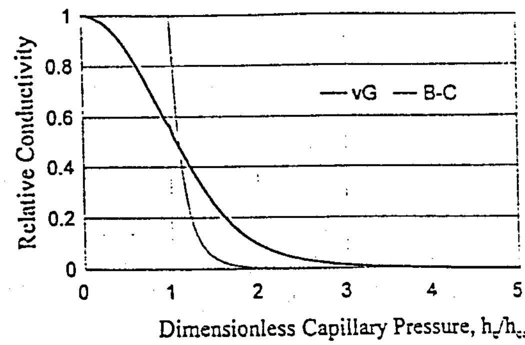

Figure 2. Relative permeability as a function of dimensionless capillary pressure head, h_c/h_{ce}, for the case $p = 4$.

Parameter Equivalence

Definition

The equivalence between the BC and vG parameters is defined based on two criteria. Primarily, the effective capillary drive must be preserved. Secondarily, the asymptotic behavior of capillary pressure versus water content should be preserved at low water contents or in other words the functional dependence on θ^* as θ^* approaches zero should be the same. This second condition, that the slope of the logarithm of the h_c curves defined by (6) and (9) should be the same, requires, as shown readily by simple asymptotic expansion, that the parameters p and m be related by the formula,

$$m = 2/(p - 1) \quad (16a)$$

or

$$p = 1 + (2/m) \quad (16b)$$

Usage

If the vG parameters α and m are known, then p can be calculated from (16b) and then h_{ce} can be calculated from the formula,

$$h_{ce} = \left(\frac{1}{\alpha}\right)\frac{(p+3)}{2p(p-1)}\left(\frac{147.8 + 8.1p + 0.092p^2}{55.6 + 7.4p + p^2}\right) \quad (17)$$

The value of H_{cM} is then deduced from (14a).

Vice versa, if the BC parameters are known, then m can be calculated from (16a), and α^{-1} can be calculated from the formula,

$$1/\alpha = h_{ce}\frac{[2p(p-1)]}{p+3}\left(\frac{55.6 + 7.4p + p^2}{147.8 + 8.1p + 0.092p^2}\right) \quad (18)$$

Figure 1 gives a visual display of (18). It shows that as p increases and soil texture and structure depart more and more from a sand and from a uniform particle size distribution [*Rawls and Brakensiek*, 1989], α^{-1} can easily be 5 to 20 times greater than h_{ce} and even more for a clay (p of the order of 20 to 25), of the order of 50.

Numerical illustration—BC parameters known. Say $p = 4$, which is a value appropriate for a sand. Since what is given by the formulae is really α^{-1}/h_{ce}, which is dimensionless, the value of h_{ce} can be taken as unity. One then calculates easily H_{cM} from (14a), yielding $H_{cM} = 1.143 \times 1$ or in other words $H_{cM}/h_{ce} = 1.143$. From (16a) it follows that $m = 0.667$. Application of (18) yields $\alpha^{-1} = 1.91h_{ce}$ or $\alpha h_{ce} = 0.52$.

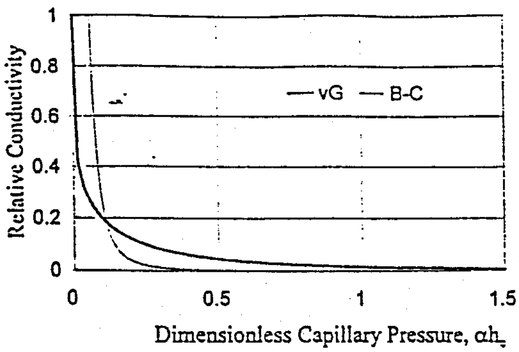

Figure 3. Relative permeability as a function of dimensionless capillary pressure head, αh_c, for the case $m = 0.2$ thus $p = 11$.

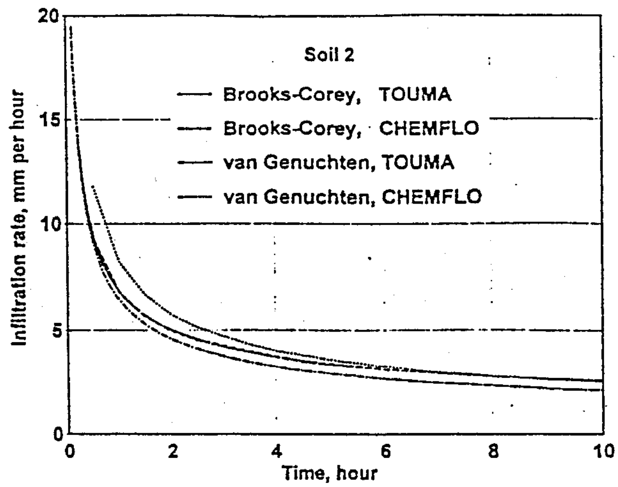

Figure 4. Infiltration rate as a function of time for soil number 2 in Table 1 using the Brooks-Corey (BC) or van Genuchten (vG) expressions according to the proposed equivalence and as calculated with the two numerical codes, CHEMFLO and *Touma*'s [1984].

The reader should note that α^{-1} is quite different from h_{ce}, in fact, about twice as large. Figure 2 shows plots of k_{rw} versus h_c, expressed in units of h_{ce} or in other words versus the dimensionless variable h_c/h_{ce}. The curves are quite different, but because H_{cM} was preserved, the areas under the two curves are equal as can be assessed visually.

Numerical illustration—VG parameters known. Say $m = 0.2$ (thus $p = 11$) and $\alpha^{-1} = 1$. Equation (17) gives $h_{ce} = 0.061 \times 1$ or $h_{ce} = 0.061\alpha^{-1}$. Now h_{ce} is only a very small fraction of α^{-1}. Vice versa, if h_{ce} is taken as unity, α^{-1} has value 16.35×1 (or $\alpha^{-1} = 16.35h_{ce}$), the value that one would read from Figure 1 for $p = 11$. Quite clearly, one cannot assume that α^{-1} and h_{ce} have the same value. Figure 3 displays the curves of relative permeability as a function of capillary pressure, expressed in units of α^{-1} or in other words as a function of the dimensionless capillary pressure variable αh_c. Again, the curves are quite different, but because H_{cM} was preserved, the areas under the two curves are equal.

Numerical Verification

We stated earlier that H_{cM}, the maximum effective capillary drive, was the important characteristic to preserve and that, if preserved, prediction of infiltration would hardly be affected by the choice of representation of the soil hydrologic properties. Several numerical codes were used to predict the infiltration capacity of a soil column for $p = 4$, 11, and 20. The value of H_{cM} was taken in the three cases to have value 40 cm. Hydraulic conductivity at natural saturation had value 1 mm per hour. The value of $\bar{\theta}$ was taken as 0.3, that of θ_r was taken as 0.05. For the uniform initial water content θ_i a value of 0.1 was selected. Table 1 shows the corresponding values of the parameters. The boundary condition at the soil surface was one of zero capillary pressure. The soil column is assumed semi-infinite.

Figures 4 and 5 display the comparison of infiltration rates and cumulative infiltration depth, respectively, versus time for soil 2 using the BC and vG expressions and two numerical codes [*Nofziger et al.*, 1989; *Touma*, 1984]. It is clear that the results are essentially unaffected by the choice of the expressions or the choice of the numerical code utilized. However, it is interesting to note that the differences between codes, especially with the vG expressions, are almost of the same order of magnitude as that resulting from the choice of expressions for the soil properties using a given code. On the other hand, Figure 6 displays the same comparison for soil 2 for the same $m = 0.2$ and $\alpha^{-1} = 416$ cm but using *Lenhard et al.*'s [1989] equivalence, which in this case yields $p = 11.26$ and $h_{ce} = 345$ cm thus $H_{cM} = 545$ cm. This latter value of 545 cm is clearly vastly different from 40 cm given in Table 1. From the dependence of infiltration rate at early time on the square root of H_{cM} in (3) one would expect the early infiltration rates to be

Table 1. Table of Correspondence for the BC and vG Parameters and Relation to Effective Capillary Drive for Three Hypothetical Soils

Soil	p	M	H_{cM}, cm	h_{ce}, cm	m	n	α^{-1}, cm	α^{-1}/H_{cM}
1	4	0.5	40	35	0.667	3	67	1.67
2	11	4	40	25.4	0.200	1.25	416	10.4
3	20	8.5	40	23	0.105	1.12	1323	33.1

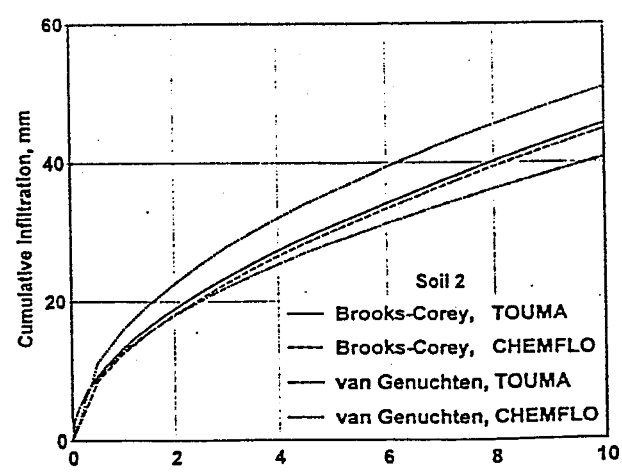

Figure 5. Cumulative infiltration depth as a function of time for soil number 2 in Table 1, using the BC or vG expressions according to the proposed equivalence and as calculated with the two numerical codes, CHEMFLO and *Touma*'s [1984].

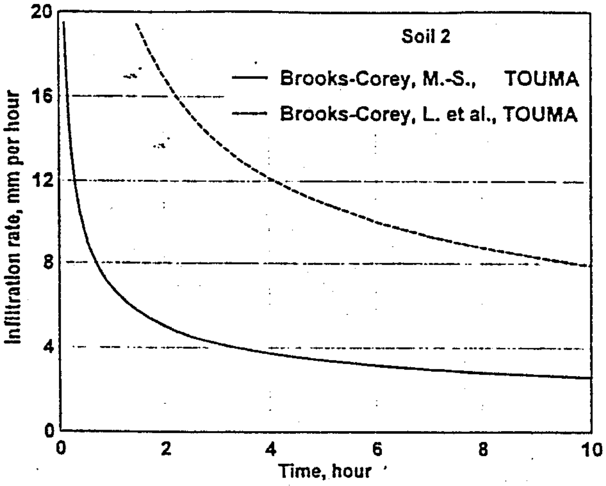

Figure 6. Infiltration rate as a function of time for soil number 2 in Table 1, using the BC expressions according to the proposed equivalence, M-S (this study), and that of *Lenhard et al.* [1989], and calculated with *Touma*'s [1984] numerical code.

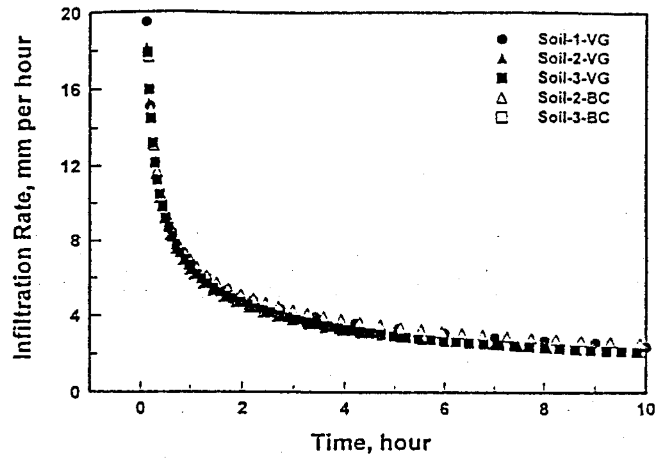

Figure 7. Infiltration rate as a function of time for soil numbers 1, 2, and 3 in Table 1, using the BC and vG expressions according to the proposed equivalence and calculated with *van Genuchten*'s [1980] numerical code.

roughly in the ratio $\sqrt{545/40} = 3.7$. Indeed, the curves look quite different, and the values are roughly in that ratio. On the other hand, with the proposed equivalence the normalized infiltration rate curves with either the BC or the vG expressions are very similar between soils 1, 2, and 3.

In Figure 7, using a van Genuchten code for the same boundary conditions and parameters and soils 1, 2, and 3 of Table 1, with the same values of H_{cM}, $\bar{K}$, $\bar{\theta}$, θ_r, and θ_i, the only difference in results is due to the difference in the p or m values, or in other words in the shape of the water content profiles. Clearly, from Figure 7 such a shape has little influence on the infiltration rates, which confirms the fact that the parameters β and b are practically constant regardless of soil texture or structure. This follows from the fact that the difference in sorptivity as given in (4) or (3), and consequently, the difference in infiltration rate is due to the parameter b or β, and since the infiltration rates are essentially the same, regardless of the value of parameter p, this implies that b or β are practically constant for all soils.

To further test the importance of the equivalence procedure on the results of infiltration calculations, three of the soils used by *Lenhard et al.* [1989] to illustrate their procedure were studied. For given known vG parameters one determines the corresponding BC curves by the two procedures, labeled M-S (this study) and L. et al. (*Lenhard et al.* [1989]), respectively, and one compares them with the vG procedure. Figures 8a, 8b, and 8c show saturation versus tension for the three soils using the two equivalences. It is clear that the Lenhard et al. procedure does a better job at matching the vG curve for each soil. Intuitively, one might think that because the h_c curves look more alike with the Lenhard et al. procedure that it would lead to better infiltration predictions. This is a case where unsupported intuition fails. Figures 9a, 9b, and 9c show relative permeability versus tension for the three soils. The M-S procedure gives a closer comparison, and it is very clear that the Lenhard et al. procedure does not preserve the area under the curve. For both soils 1 and 4 the areas are quite different. As expected, Figures 10a, 10b, and 10c show greatly different curves of infiltration rates and cumulative infiltration with the Lenhard et al. procedure. The better performance of the proposed procedure is understandable because $\rho_w g H_{cM}$, where ρ_w is density of water and g is acceleration of gravity, is the energy per unit volume expanded by the capillary forces during the flow of water from a region of a high given saturation (or low capillary pressure) to a region of a different (low) satura-

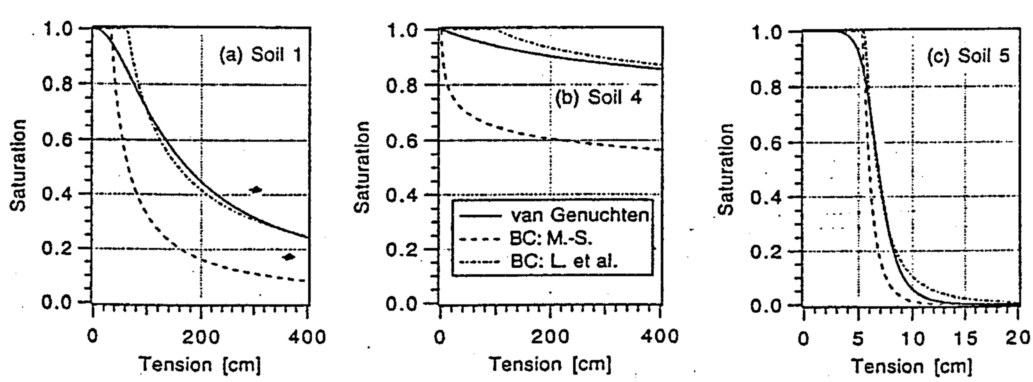

Figure 8. Saturation as a function of tension (centimeters) for soil numbers (a) 1, (b) 4, and (c) 5 used in *Lenhard et al.*'s [1989] paper, using the vG expressions and the BC expressions according to the proposed M-S equivalence and the Lenhard et al. equivalence.

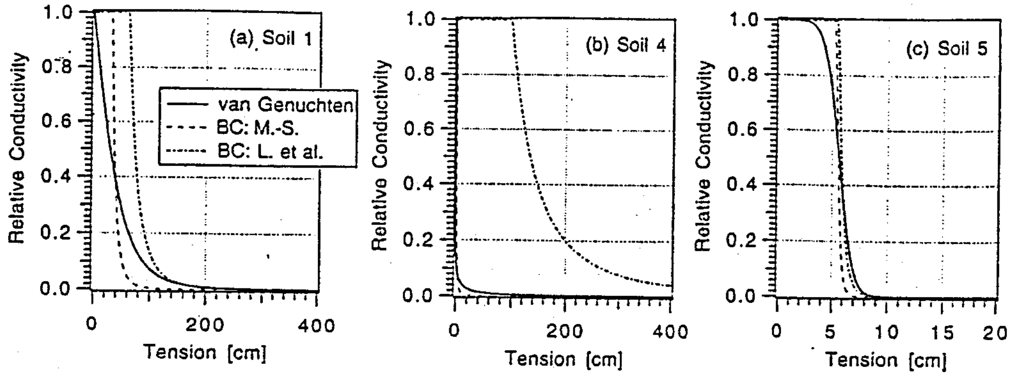

Figure 9. Relative permeability as a function of tension (centimeters) for soil numbers (a) 1, (b) 4, and (c) 5 used in Lenhard et al.'s [1989] paper, using the vG expressions and the BC expressions according to the proposed M-S equivalence and the Lenhard et al. equivalence.

tion. H_{cM} is a dynamic capillary concept, whereas the $\theta^*(h_c)$ curve is a static one. H_{cM} is a potential in the traditional fluid mechanics sense as shown by *Morel-Seytoux and Khanji* [1974]. The value of H_{cM} between the maximum and the minimum values of saturation is independent of the shape of the saturation profile between these two values. Figures 11a, 11b, and 11c show profiles of water contents at time 1 day for the three soils. The profiles match well with the M-S procedure and are very different with the *Lenhard et al.* [1989] procedure. The numerical calculations for these three examples were carried out with the code STOMP [*White et al.*, 1992].

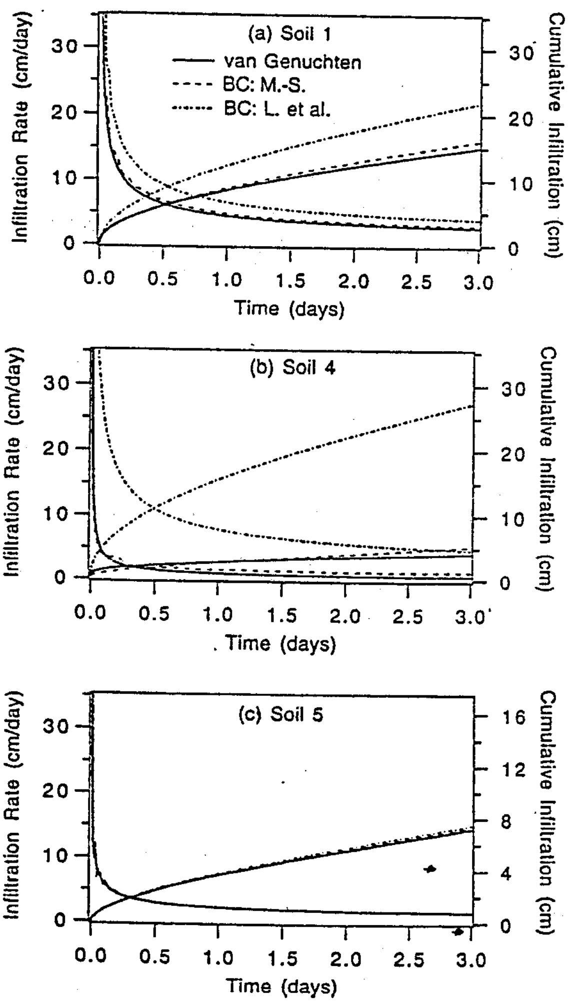

Figure 10. Infiltration rate and cumulative infiltration depth as a function of time for soil numbers (a) 1, (b) 4, and (c) 5 used in *Lenhard et al.*'s [1989] paper, using the vG expressions and the BC expressions according to the proposed M-S equivalence and the Lenhard et al. equivalence and calculated with the numerical code STOMP.

Influence of Initial Conditions

It may be worthwhile to mention that using H_{cM} in (2) implies that the initial value of water content is low to moderate. Otherwise, one would have to use the more general definition of the effective capillary drive $H_c(\theta_i)$:

$$H_c(\theta_i) = \int_0^{h_{ci}} k_{rw}\, dh_c = H_{cM}\left(1 - \frac{M}{p}\,\theta_i^{*p-M}\right) \tag{19a}$$

or

$$H_c(\theta_i) = H_{cM}\left[1 - \frac{M}{p}\left(\frac{h_{ci}}{h_{ce}}\right)^{-(p/M)+1}\right] \tag{19b}$$

where h_{ci} is h_c for $\theta = \theta_i$, p and M being related by (8). One can verify that the deviation from H_{cM} is significant only for relatively large values of θ_i^*. For $p = 5$ thus $M = 1$ the normalized initial water content would have to be 0.84 for it to have a 10% influence on the value of the effective capillary drive. For $p = 11$ thus $M = 4$ the value of θ_i^* would have to be 0.83. For $\theta_i^* = 0.5$ the percentage of decrease in effective capillary drive would be 1% for $p = 5$ and 0.3% for $p = 11$. Thus it is clear that in the low to moderate range of water contents, the initial value of the water content has very little impact on the effective capillary drive. In that range of initial values the preservation of the maximum value of the effective capillary drive will guarantee the insensitivity of the model used to represent the soil hydraulic properties on the calculations of infiltration capacity. Of course, the value of the infiltration rate will change but practically entirely from the contribution of the square root of $(\bar{\theta} - \theta_i)$ which appears in the expression for the infiltration rate in (2) and (3). If, on the other hand, the initial water contents are very high and given that at the boundary under ponded conditions saturation holds, then capillary drive is not a force; sorptivity is essentially zero and equivalence is not important. Gravity dominates the flow. The equivalence is important when in some regions water

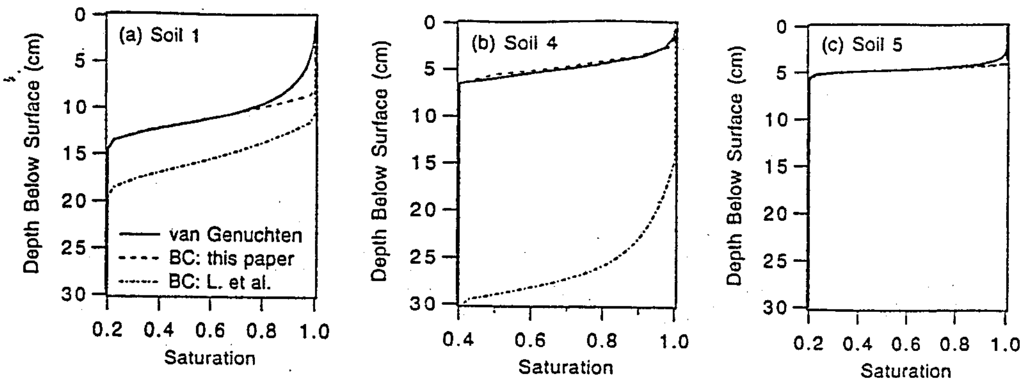

Figure 11. a, b and c. Water content profiles at time 1 day for soil numbers 1, 4, and 5 used in *Lenhard et al.*'s [1989] paper, using the vG expressions and the BC expressions according to the proposed M-S equivalence and the Lenhard et al. equivalence and calculated with the numerical code STOMP.

contents are low to moderate and in others they are at or near saturation. This is the case that was investigated which is crucial for watershed rainfall-runoff modeling, where saturation under rainfall occurs at the soil surface, whereas below the soil is initially dry, and in capillary barrier design, where saturation occurs at the interface between the coarse material and the fine material above it.

Conclusions

An equivalence was defined which provides a simple way to convert BC parameters into vG parameters and vice versa, when one set is known and preserves the value of the effective capillary drive thus making infiltration capacity calculations insensitive to the model used to represent the soil hydraulic properties. It is strictly then a matter of convenience for the user which expression is used. Naturally, this conclusion applies only for situations which lead to high water contents in some part of the domain of concern. Because the second criterion for the equivalence preserves the asymptotic behavior of capillary pressure at low water contents and thus the hydraulic gradient, there is at least plausible ground that the equivalence will be reasonably satisfactory in that range also. However, this should be tested for situations involving drainage and evapotranspiration. It is recommended that it be done before acceptance for use in these situations. Finally, other equivalences should be developed between other types of expressions, particularly three-parameter ones.

Appendix

The numerical evaluation of the effective capillary drive using the van Genuchten functions for the soil capillary and hydraulic properties follows. The dimensionless effective capillary drive, defined as

$$\mathrm{SUM} = \alpha \int_0^\infty k_{rw}(h_c)\, d\vec{h}_c \qquad \text{(A1)}$$

was evaluated numerically by Gaussian quadrature using 10, 20, 60, 120, and 256 Gauss points. Results for 120 and 256 Gauss points were essentially identical. A rational function was fitted by least squares to the results of the numerical integration using 256 Gauss points and the optimized expression is

$$\mathrm{SUM} = \frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2} \qquad \text{(A2)}$$

Relative deviation percentages are always less than 0.5% for m values between $m = 0.7$ (where SUM = 0.64) and $m = 0.2$ (where SUM = 0.1) and 1.6% for m values between 0.2 and 0.05 (where SUM = 0.0078). The practical range of m is roughly between 0.67 and 0.05, corresponding to a range of p between 4 and 40. Thus, within that range the accuracy of the rational expression is quite sufficient. The differences in the results for soils 1, 2, and 3 of Table 1 using the vG or BC expressions are sufficiently small that part of the difference may have come from these small errors, but from a practical point of view that has essentially no impact on the merit of the proposed equivalence.

Acknowledgments. The investigation was carried out while the first author was on an Associated Western Universities Fellowship at Pacific Northwest Laboratory in spring 1994. Financial assistance from the U.S. Nuclear Regulatory Commission, Office of Nuclear Regulatory Research is appreciated. Thanks are extended to Mark Rockhold for many useful discussions and suggestions in the preparation of the manuscript. The first version of this paper was submitted on November 17, 1994 and acknowledged as received by the WRR office on November 28, 1994 as WR94-733.

References

Brooks, R. J., and A. T. Corey, Hydraulic properties of porous media, *Hydrol. Pap. 3*, Colo. State Univ., Fort Collins, 1964.

Corey, A., *Mechanics of heterogeneous fluids in porous media*, 150 pp., Water Resour., Publ., Fort Collins, Colo., 1977.

Lenhard, R. J., J. C. Parker, and S. Mishra, On the correspondence between Brooks-Corey and van Genuchten Models, *J. Irrig. Drain. Eng., 115*(4), 744–751, 1989.

Morel-Seytoux, H. J., Introduction to flow of immiscible liquids in porous media, in *Flow Through Porous Media*, edited by R. deWiest, chap. XI, pp. 455–516, Academic, San Diego, Calif., 1969.

Morel-Seytoux, H. J., and J. A. Billica, A two-phase numerical model for prediction of infiltration: Applications to a semi-infinite soil column, *Water Resour. Res., 21*(4), 607–615, 1985.

Morel-Seytoux, H. J., and J. Khanji, Derivation of an equation of infiltration, *Water Resour. Res., 10*(4), 795–800, 1974.

Morel-Seytoux, H. J., and J. Khanji, Prediction of imbibition in a horizontal column, *Soil Sci. Soc. Am. Proc., 39*(4), 613–617, 1975.

Nofziger, D. L., K. Rajender, S. K. Nayudu, and P.-Y. Su, CHEMFLO: One-dimensional water and chemical movement in unsaturated soils, *Rep. EPA/600/8-89/076*, Robert S. Kerr Environ. Res. Lab., Ada, Okla., 1989.

Philip, J. R., Reply to "Comments on steady infiltration from spherical cavities," *Soil Sci. Soc. Am. J.*, *49*, 788–789, 1985.
Rawls, W. J., and D. L. Brakensiek, Estimation of soil water retention and hydraulic properties, in *Unsaturated Flow in Hydrologic Modeling*, edited by H. J. Morel-Seytoux, pp. 275–300, Kluwer Acad., Norwell, Mass., 1989.
Russo, D., E. Bresler, U. Shani, and J. C. Parker, Analyses of infiltration events in relation to determining soil hydraulic properties by inverse problem methodology, *Water Resour. Res.*, *27*(6), 1361–1373, 1991.
Touma, J., Etude critique de la caractérisation hydrodynamique des sols non saturés: Rôle de l'air; influence de l'écoulement multidimensionnel de l'eau, Ph.D. thesis, 190 pp., Sci. and Med. Univ. of Grenoble, Natl. Polytech. Inst. of Grenoble, Grenoble, France, 1984.
van Genuchten, M. T., A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am.*, *44*, 892–898, 1980.
Warrick, A. W., and P. Broadridge, Sorptivity and macroscopic capillary length relationships, *Water Resour. Res.*, *28*(2), 427–431, 1992.
White, I., and M. J. Sully, Macroscopic and microscopic capillary length scale and timescales from field infiltration, *Water Resour. Res.*, *23*(8), 1514–1522, 1987.
White, M. D., R. J. Lenhard, W. A. Perkins, and K. R. Roberson, Arid-id engineering simulator design document, *PNL-8448*, Pac. Northwest Lab., Richland, Wash., 1992.

R. J. Lenhard and P. Meyer, Pacific Northwest Laboratories, Richland, WA 99352. (e-mail: pd_meyer@ccmail.pnl.gov)
H. J. Morel-Seytoux, Hydrology Days Publications, 57 Selby Lane, Atherton, CA 94027-3926. (e-mail: morelsey@leland.stanford.edu)
M. Nachabe, Department of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder, CO 80309. (e-mail: nachabe@gpsr.colostate.edu)
J. Touma, Département des Eaux Continentales, Institut Français de Recherche Scientifique pour le Développement en Coopération (ORSTOM), 34032 Montpellier, France. (e-mail: touma@rio.net)
M. T. van Genuchten, U.S. Salinity Laboratory, USDA Agricultural Research Service, 4500 Glenwood Drive, Riverside, CA 92507. (e-mail: vangenu@ito.umnw.ethz.ch)

(Received May 22, 1995; revised January 2, 1996; accepted January 4, 1996.)