INFILTRATION OVER HETEROGENOUS WATERSHED: INFLUENCE OF RAIN EXCESS

By Mahmood H. Nachabe,[1] Tissa H. Illangasekare,[2] Hubert J. Morel-Seytoux,[3] Laj R. Ahuja,[4] Members, ASCE, and Huan Ruan[5]

ABSTRACT: Once the soil surface reaches saturation, rain excess is generated. This rain excess may run over and infiltrate in other areas of the watershed. In this study, we consider three scenarios of distributing rain excess/run-on to examine how it affects infiltration over a watershed. For these scenarios, we introduce simple solutions that allow the users to (1) estimate the ponded area-fraction of the watershed, and (2) determine the area-averaged infiltration. The solutions are practical because they require trivial computations. The solutions of ponding time and area-averaged infiltration account for the influence of initial water content and contain parameters with precise physical meaning. We find that area-averaged infiltration depends strongly on the distribution of rain excess at the surface. Thus we question the use of many stochastic area-averaged infiltration models that ignore the influence of rain excess.

INTRODUCTION

Reliable and practical models of large-scale infiltration are important in watershed management, hydrology, agriculture, and climatology. Infiltration at a local or measurement scale integrates soil-water characteristics into meaningful parameters like sorptivity and saturated hydraulic conductivity. In general, these parameters vary spatially over a watershed and we are interested in including the knowledge of this variability in predicting large-scale area-averaged infiltration. In this study, we introduce simple models to predict ponding time and area-averaged infiltration over a heterogeneous watershed.

At the local scale, parameters of the infiltration equations, e.g. Green and Ampt or Philip infiltration equations, include the hydraulic conductivity at natural saturation (K_s) and the effective capillary drive (H) or the sorptivity (S). The sorptivity can be expressed in terms of K_s using the simple algebraic equation (White and Sully 1987)

$$S^2 = \frac{H\Delta\theta K_s}{b} \quad (1)$$

where b = parameter close to 0.55 for most field soils and $\Delta\theta$ = difference between saturated and initial soil-water content. The effective capillary drive can be expressed well as a function of the relative conductivity as (Morel-Seytoux et al. 1996)

$$H = \int_{-\infty}^{0} k_w(\psi)\, d\psi \quad (2)$$

where $k_w(\psi)$ = relative conductivity of water as a function of water pressure, ψ.

In passing from the local scale to an area-averaged infiltration model, measurements of conductivity are assumed available at selected locations over the heterogeneous watershed. Hence, given a set of spatially distributed measurements of hydraulic conductivity, one constructs the areal frequency distribution function of K_s, i.e. $f(K_s)$. The knowledge of $f(K_s)$ shall be used to develop the area-averaged infiltration over the watershed.

If a zone of the watershed with relatively low conductivity reaches ponding, rain excess may spill over from the ponded zone and infiltrates in the nonponded zone. Existing stochastic models developed by Maller and Sharma (1981), Dagan and Bresler (1983), Montoglou and Gelhar (1987), and Chen et al. (1994), among others, ignored the influence of spillover of rain excess on area-averaged infiltration. In this study, we analyze three scenarios of distributing the rain excess at the surface to investigate its role in predicting area-averaged infiltration.

THEORY AND ASSUMPTIONS

The first step in modeling area-averaged infiltration is to select an infiltration capacity equation. This equation is applicable after ponding because all applied water infiltrates prior to saturation of the soil's surface.

Philip's Infiltration Equation

We adopt Philip's infiltration equation in this study. Philip's equation has been used in numerous field studies because it is simple and contains parameters with precise physical meaning (Singh 1988; Philip 1969). The infiltration capacity at a local scale is assumed to be

$$I_c(t) = \frac{1}{2} S t^{-1/2} + K_s \quad (3)$$

where t = time assuming instantaneous ponding. The sorptivity in (3) can be expressed in terms of K_s using (1) to obtain

$$I_c(t) = \left(\frac{H\Delta\theta K_s}{b}\right)^{1/2} t^{-1/2} + K_s \quad (4)$$

Eq. (4) shows the dependence of infiltration capacity on initial soil-water content and hydraulic conductivity. The first term on the right hand of (4) represents the ability of the soil to infiltrate water by capillarity. For an initially dry soil, $\Delta\theta$ is large and this term is important during the early stage of infiltration, as expected. At later times, the hydraulic gradient at the surface is close to unity. At this stage, the infiltration rate is equal to the saturated conductivity because the flow becomes dominated essentially by gravity.

Philip's infiltration solution, which was derived for a prescribed water content boundary condition at the surface has been extended for a rainfall boundary condition by Maller and

[1]Water Engrg. and Mgmt. Program, Asian Inst. of Technol., P.O. Box 4, Klongluang, Pathum Thani 12120, Thailand; formerly, U.S. Dept. of Agr., Agr. Res. Services, 301 S. Howes, Fort Collins, CO 80522.
[2]Prof., Univ. of Colorado at Boulder, CB 428, Boulder, CO 80309-0428.
[3]Hydrology Days Publication, 57 Selby Lane, Atherton, CA 94027.
[4]Res. Leader, U.S. Dept. of Agr., Agr. Res. Services, 301 S. Howes, Fort Collins, CO.
[5]Grad. Student, Univ. of Colorado at Boulder, CB 428, Boulder, CO.
Note. Discussion open until December 1, 1997. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this technical note was submitted for review and possible publication on October 25, 1994. This technical note is part of the *Journal of Hydrologic Engineering*, Vol. 2, No. 3, July, 1997. ©ASCE, ISSN 1084-0699/97/0003-0140–0143/$4.00 + $.50 per page. Technical Note No. 9471.

Sharma (1981) and Flemming and Smiles (1975). For a rainfall boundary condition, an equivalent ponding time (t_e) is first defined by setting (3) equal to the rainfall rate (r). Ponding is assumed to occur at time t_p such that the cumulative infiltration up to t_e is equal to the applied rain. Mathematically, $t_e = \{S/[2(r - K_s)]\}^2$ and $t_p = [(2r - K_s)/r]t_e$ (Maller and Sharma 1981). Before t_p, the infiltration rate is equal to the rainfall rate. After t_p, the infiltration rate is given by Philip's equation with the time origin shifted. Accordingly, the infiltration rate for a rainfall boundary condition is

$$I(t) = r, \quad 0 \le t \le t_p \tag{5a}$$

$$I(t) = \frac{1}{2}\left(\frac{H\Delta\theta K_s}{b}\right)^{1/2} [t - (t_p - t_e)]^{-1/2} + K_s, \quad t \ge t_p \tag{5b}$$

This extended Philip solution is an approximate solution, and was derived for a constant rainfall rate. The influence of the initial soil-water content and K_s on the ponding time are demonstrated by substituting the sorptivity by its equivalent in (1). Symbolically, the ponding time is

$$t_p = \frac{H\Delta\theta K_s}{r(r - K_s)}\left[\frac{2r - K_s}{4b(r - K_s)}\right] \tag{6}$$

If the soil is initially saturated, $\Delta\theta$ at the surface is zero and ponding is instantaneous, i.e. $t_p = 0$. The drier the soil, the longer is the time to ponding. Ponding will not occur at any time unless the rainfall rate exceeds K_s. Finally, t_p increases monotonically with K_s indicating that less permeable soils reach ponding faster than more permeable soils in a heterogenous watershed.

Variability of Hydraulic Conductivity

The hydraulic conductivity, K_s, may exhibit orders of magnitude of variation over a heterogeneous watershed. Rawls and Brakensiek (1988) suggested that K_s ranges between 0.001 cm/hr^{-1} for clay to over 10 cm/hr^{-1} for a coarse sandy soil. We hypothesize, following most investigators, that $f(K_s)$, the spatial frequency distribution of K_s is lognormally distributed. Hence, $f(K_s)$ is well represented by (Yevjevich 1972)

$$f(K_s) = \frac{1}{K_s(2\pi)^{1/2}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\ln K_s - \mu}{\sigma}\right)^2\right] \tag{7}$$

where μ and σ are the mean and standard deviation of the logarithm of K_s. The geometric mean is $K_G = \exp(\mu)$ and the arithmetic mean is $K_A = \exp(\mu + \sigma^2/2)$. The frequency distribution of K_s reflects the areal distribution of soils over the heterogeneous watershed. For example, the area under the lower tail of $f(K_s)$ represents a fraction-area of the watershed with clayey soil with low conductivity, whereas the area under the upper tail of $f(K_s)$ may represent a fraction-area of the watershed with coarse sandy soils, or macropores. For the lognormal distribution, $\sigma = 1$ indicates that K_s varies within four orders of magnitude in 95.4% of the watershed area. Mathematically, the cumulative distribution

$$F(K_s) = \int_0^{K_s} f(u)\, du \tag{8}$$

is the area-fraction of the watershed with saturated conductivity less than K_s.

Spillover from Ponded to Nonponded Zone

Spillover and distribution of rain excess are governed essentially by topography (surface elevation). The distribution of rain excess will influence the area-averaged infiltration by controlling the supply rate at the surface. Three scenarios of K_s distribution with topography are investigated here. In the first scenario, we consider K_s to increase monotonically with increase in elevation. The conductivity was found to increase monotonically over tropical catenas with moderate slopes because surface runoff carries fine particles to lower elevations (Mapa 1994). For this condition, the lower zone of the watershed reaches ponding first (because of the low conductivity) and ponding propagates upward with time [see Fig. 1(a)]. Hence, excess rain cannot contribute to the nonponded zone upslope, and the supply rate over the nonponded zone will not exceed the rainfall rate. Similar assumptions are inherent in the stochastic models of Maller and Sharma (1981), Dagan and Bresler (1983), and Chen et al. (1994) that ignored the influence of rain excess.

The variation of K_s in the second scenario is the opposite of scenario one. The hydraulic conductivity decreases with the increase in surface elevation. The upper zone of the watershed reaches ponding first, and a ponding front propagates downward [Fig. 1(b)]. For this scenario, the ponded-area-fraction is determined by the condition that its mean infiltration rate is equal to the rainfall rate.

Finally, in the third scenario, K_s is considered to vary arbitrarily over a flat surface. This condition may prevail in certain leveled agricultural fields. In this scenario, we assume uniform spillover from the ponded zone to the nonponded zone. The supply rate over the nonponded zone (r_s) is equal to the rainfall rate plus the excess rain from the ponded zone.

In all scenarios, detention storage is neglected. Also, because the ponded depth is relatively small, we neglect its influence on the infiltration capacity for a soil, i.e., (5), applies after ponding. Finally, we assume a uniform and constant rainfall rate over the watershed in the following derivations.

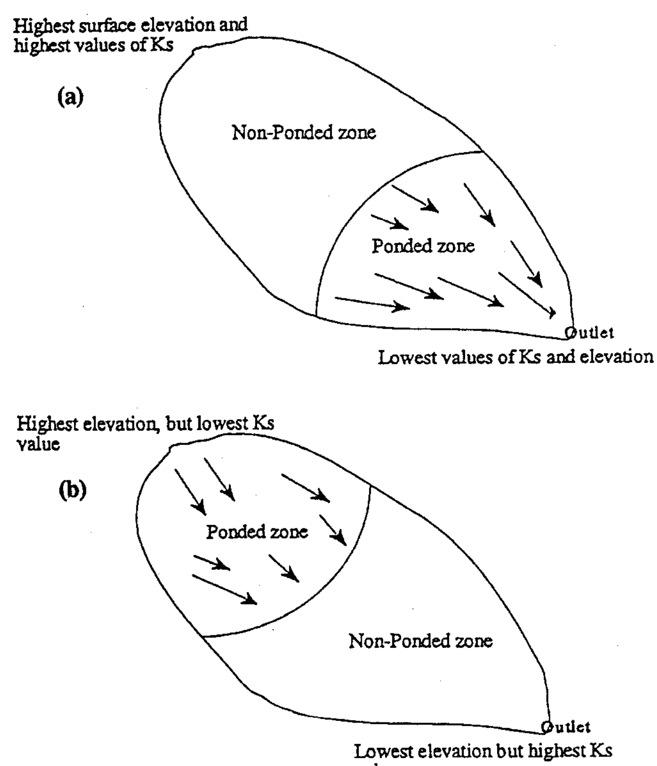

FIG. 1. Arrows Show Direction of Flow of Rain Excess: (a) Saturated Conductivity Is Assumed to Increase Monotonically with Elevation, Thus There Will Be No Spill from Ponded to Nonponded Zone; (b) Opposite of (a)

METHODS

Scenario I: No Spilling from Ponded to Nonponded Zone

At time t from the start of rain, ponding will occur in the fraction of the watershed that has $t_p < t$. From (6), this will correspond to the area-fraction having a conductivity $K_s < r\{1 - [1 - t/(t + a)]^{1/2}\}$ where $a = H\Delta\theta/4br$, a constant. Hence, the evolution with time of the ponded fraction of the watershed $[F_p(t)]$ is

$$F_p(t) = \int_0^{r\{1-[1-(t/t+a)]^{1/2}\}} f(K_s)\, dK_s \qquad (9)$$

The mean infiltration rate over this ponded fraction is

$$I_{cp}(t) = \frac{\int_0^{r\{1-[1-(t/t+a)]^{1/2}\}} I_c(K_s, t) f(K_s)\, dK_s}{F_p(t)} \qquad (10)$$

where $I_c(K_s, t)$ is the local infiltration capacity in (5). Because there is no spill of rain excess, the supply rate over the nonponded fraction of the watershed is equal to the rainfall rate. Hence, the area-averaged infiltration rate $[I_A(t)]$ at time t is

$$I_A(t) = r[1 - F_p(t)] + I_{cp}[F_p(t)] \qquad (11)$$

The first and second terms on the right side of (11) represent the contribution of the nonponded and ponded fractions to area-averaged infiltration.

Scenario II: K_s Decreasing Monotonically with Elevation

In this scenario, the upper zone of the watershed reaches ponding first and the rain excess will flood an adjacent area downslope. Unlike scenario I, this rain excess will accelerate the progress of ponding in the watershed. The ponded area-fraction of the watershed is determined by the condition that the mean infiltration rate over the ponded zone is equal to the rainfall rate. Mathematically

$$I_{cp}(t) = \frac{\int_0^{r_m\{1-[1-(t/t+a)]^{1/2}\}} I_c(K_s, t) f(K_s)\, dK_s}{F_p(t)} = r \qquad (12)$$

where

$$F_p(t) = \int_0^{r_m\{1-[1-(t/t+a)]^{1/2}\}} f(K_s)\, dK_s \qquad (13)$$

and r_m is the maximum supply rate at which ponding occurs. To determine $F_p(t)$, the ponded area-fraction of the watershed, (12) is solved for the unknown r_m as a function of time t. The area-averaged infiltration, $I_A(t)$, is given by (11), but because of the condition in (12), $I_A(t)$ is equivalent to the rainfall rate. If the duration of rain is long enough, the entire watershed reaches ponding asymptotically at time T. For practical purposes in this study, T is the time at which $F_p(T) = 0.99$. After time T, the area-averaged infiltration rate progresses at its capacity,

$$I_A(t) = I_{cp}(t) = \int_0^{r_m\{1-[1-(t/t+a)]^{1/2}\}} I_c(K_s, t) f(K_s)\, dK_s \qquad (14)$$

Scenario III: Uniform Distribution of Rain Excess over Nonponded Zone

In this scenario, there is spillover from the ponded zone to the nonponded zone. We assume that this rain excess is uniformly distributed over the nonponded zone. Thus over the nonponded zone, the water supply rate at the surface (r_s) is the rainfall rate (r) plus the rain excess from the ponded zone. Mathematically, r_s is

$$r_s = r + \frac{F_p}{1 - F_p}(r - I_{cp}) \qquad (15)$$

where $F_p(t)$ is the ponded fraction of the watershed

$$F_p(t) = \int_0^{r_s\{1-[1-(t/t+a)]^{1/2}\}} f(K_s)\, dK_s \qquad (16)$$

and $I_{cp}(t)$ is the mean infiltration rate of this fraction expressed as

$$I_{cp}(t) = \frac{\int_0^{r_s\{1-[1-(t/t+a)]^{1/2}\}} I_c(K_s, t) f(K_s)\, dK_s}{F_p(t)} \qquad (17)$$

Eq. (15) is solved for the unknown (r_s) as a function of time. As ponding progresses over the watershed, both F_p and r_s increase with time. Eventually, the entire watershed reaches ponding at time T. After time T, infiltration progresses at capacity.

Numerical Simulations

The integral in (9) can be evaluated from the table of integral for the normal distribution (Yevjevich 1972). The integrals in (12) and (15) should be performed numerically. The computation time to complete a simulation shall not exceed a couple of minutes on a personal microcomputer.

ILLUSTRATIONS

To demonstrate the influence of rain excess, we choose a reference simulation with parameters: $\mu = -0.5$, i.e. $K_G = 0.606$ cm/hr^{-1}; $\sigma = 1$, i.e. $K_A = 1$ cm/hr^{-1}; $r = 2$ cm/hr^{-1}; $\Delta\theta = 0.3$; and $H = 10$ cm. Figs. 2 and 3 show the evolution with time of the ponded area-fraction and the area-averaged infiltration. Ponding was fastest in scenario II. For example, after 0.5 h from start of rain, 81% of the watershed surface reaches ponding in scenario II, versus 53% and 42% for scenarios III and I, respectively. This behavior was anticipated because in scenario II all rain excess is used to achieve ponding, as required by (13). Ninety-nine percent of the watershed reached ponding at $T = 0.8$ and 1.4 h for scenarios II and III, whereas the maximum ponded area-fraction was only 82% in scenario I. As demonstrated in Fig. 3, the area-averaged infiltration for scenario III was larger than the area-averaged infiltration for

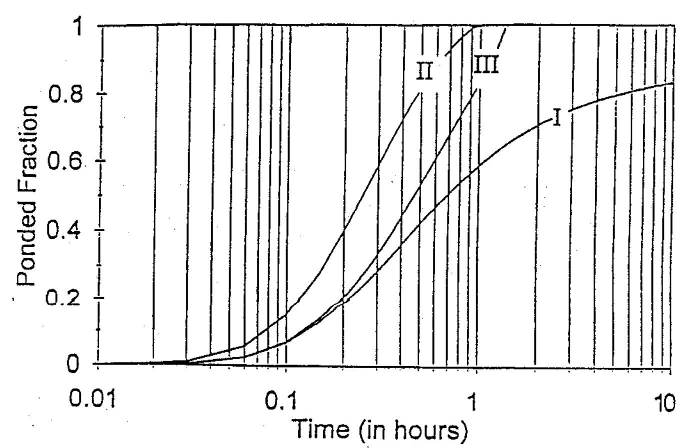

FIG. 2. Evolution with Time of Ponded Area-Fraction for Three Scenarios

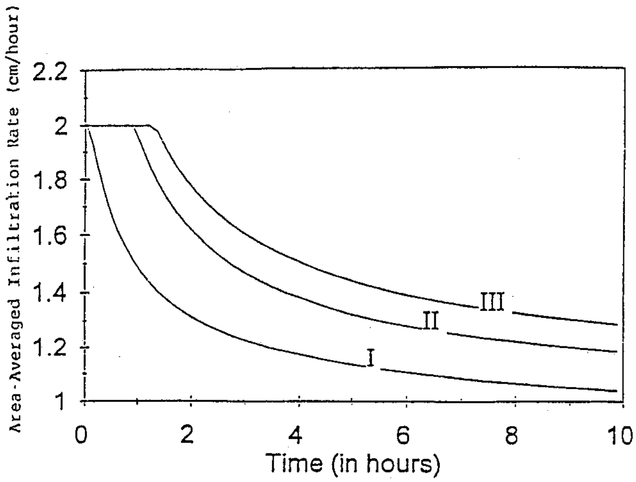

FIG. 3. Area-Averaged Infiltration for Three Scenarios

the two other scenarios. Again, these results demonstrate that the distribution of rain excess will play an important role in determining infiltration and runoff over heterogeneous surfaces. Hence, even though existing large-scale stochastic models [e.g., Maller and Sharma (1981); Dagan and Bresler (1983); Chen et al. (1994)] account for spatial variability, their applications to area-averaged infiltration neglect rain excess. This may not always provide a realistic description of field conditions as argued by the example scenarios in this study. This finding agrees with the recent work of Woolhiser et al. (1996) on the importance of trends in conductivity on the prediction of infiltration and runoff. The method of Woolhiser et al. (1996), which couples overland flow with infiltration, also assumes that the hydraulic conductivity is a linear trend. We note that a linear trend implies a uniform frequency distribution of conductivity rather than a lognormal frequency distribution.

The models presented here are simple and contain parameters with precise physical meaning that can be easily determined in the field using a disc permeameter or tension Infiltrometer [e.g. Nachabe and Illangasekare (1994)]. Also, these models account for initial soil-water condition ($\Delta\theta$) and effective capillary drive (H). Variation in these parameters can be incorporated here deterministically. Although spatial variations in H can be important, data from field studies [e.g. White and Sully (1987)] have indicated that variability in H can be small as compared to variability in K_s.

Finally, for a reader contemplating using (11), (12), (13), or (16) and (17) to predict area-averaged infiltration, we offer a word of warning. First, the extended Philip solution was developed under constant rainfall rate and we chose to use it here for variable water supply rate at the surface; assuming constant a in scenarios II and III may be inaccurate. Secondly, the actual interplay between rain excess and infiltration in a watershed can be more complicated than the three scenarios. Indeed, occasionally rain excess flows to small rills and channels in the watershed and does not run on and infiltrate in other parts of the watershed. For other surface conditions, rain excess can be retained in surface depressions where it slowly evaporates. Thus, the actual area-averaged infiltration can be anywhere between scenarios I, II, and III: scenario I allows all the generated rain excess to escape infiltration and to become runoff at the outlet; scenario II accelerates ponding over the watershed, but does not allow rain excess to infiltrate in the nonponded zone; finally, scenario III assumes rain excess feeds infiltration into the nonponded zone of the watershed.

APPENDIX I. REFERENCES

Chen, Z. Q., Govindaraju, R. S., and Kavvas, M. L. (1994). "Spatial averaging of unsaturated flow equations under infiltration conditions over areally heterogeneous fields. 1: Development of models." *Water Resour. Res.*, 30(2), 523–533.

Dagan, G., and Bresler, E. (1983). "Unsaturated flow in spatially variable fields. 1: Derivation of models of infiltration and redistribution." *Water Resour. Res.*, 19, 413–420.

Flemming, P. M., and Smiles, D. E. (1975). "Infiltration of water into soil." *Proc., 3rd Hydrol. Symp.*, 83–110.

Maller, R. A., and Sharma, M. L. (1981). "An analysis of areal infiltration considering spatial variability." *J. Hydrol.*, 52, 25–27.

Mapa, R. B. (1994). "Variation of soil hydraulic properties along an alfisol catena." *Abstr., 14th Annu. AGU Mtg.*, H. J. Morel-Seytoux, ed., Hydrology Days Publication, Atherton, Calif.

Montoglou, A., and Gelhar, L. W. (1987). "Stochastic modeling of large scale transient unsaturated flow systems." *Water Resour. Res.*, 23(1), 37–46.

Morel-Seytoux, H. J., Meyer, P., Nachabe, M. H., Touma, J., van Genuchten, R., and Lenhard, R. (1996). "Parameter equivalence for the Brooks-Corey and van Genuchten soil characteristics, preserving the effective capillary drive." *Water Resour. Res.*, 32, 1251–1258.

Nachabe, M. H., and Illangasekare, T. H. (1994). "Use of tension infiltrometer data with unsaturated hydraulic conductivity models." *Ground Water*, 32, 1017–1021.

Philip, J. R. (1969). "Theory of infiltration." *Advanced Hydrosci.*, 5, 215–296.

Rawls, W. J., and Brakensiek, D. L. (1989). "Estimation of soil water retention and hydraulic properties." *Unsaturated flow in hydrologic modeling*, H. J. Morel-Seytoux, ed., 275–300.

Singh, V. P. (1988). *Hydrologic systems in watershed modeling*, Vol. 2. Prentice-Hall, Inc., Englewood, N.J.

White, I., and Sully, M. J. (1987). "Macroscopic and microscopic capillary length and time scales from field infiltration." *Water Resour. Res.*, 23, 1514–1522.

Woolhiser, D. A., Smith, R. E., and Giraldez, J. V. (1996). "Effects of spatial variability of saturated conductivity on Hortonian overland flow." *Water Resour. Res.*, 33, 671–678.

Yevjevich, V. (1972). *Probability and statistics in hydrology*. Water Resources Publications, Littleton, Colo.

APPENDIX II. NOTATION

The following symbols are used in this paper:

b = 0.55;
$e^{(\ )}$ = $2.718281828^{(\ )}$;
$F(\ )$ = cumulative frequency distribution of variable ( );
F_p = ponded fraction of watershed;
$f(\ )$ = frequency distribution function of variable ( );
H = effective capillary drive;
I_A = area-averaged infiltration rate for watershed;
I_c = infiltration capacity at local (or measurement) scale;
I_{cp} = mean capacity infiltration rate over ponded zone;
K_s = hydraulic conductivity at natural saturation;
K_G = geometric mean of K_s;
k_w = relative permeability of water;
r = rainfall rate;
r_m = maximum supply rate over ponded zone;
r_s = supply rate over nonponded zone of surface;
S = sorptivity;
T = time at which $F_p(t)$ = 0.99; applicable for all scenarios;
t = time since beginning of rain;
t_p = local ponding time;
$\Delta\theta$ = difference between initial and saturated soil-water content;
μ = arithmetic mean of logarithm of K_s; and
σ = standard deviation of logarithm of K_s.