Use of Tension Infiltrometer Data with Unsaturated Hydraulic Conductivity Models

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Abstract

A new method is presented to estimate the relationship between unsaturated hydraulic conductivity and soil-water pressure using tension infiltrometer data. In this method, both the early transient and the subsequent quasi-steady stages of the infiltration process underneath a tension infiltrometer disc are analyzed. Our investigation indicates that the infiltration rate at early times closely follows the aquare-root-of-time relationship (Philip, 1969). This behavior permits the evaluation of the sorptivity, which is an integrated measure of soil diffusivity. At later times, three-dimensional quasi-steady flow beneath the infiltrometer disc develops, and the Wooding equation for flow from a circular pond is used to evaluate the sorptive number and the saturated hydraulic conductivity. Simple algebraic expressions are introduced to relate the sorptivity and sorptive number to the parameters of the Brooks and Corey model of unsaturated hydraulic conductivity. The method described here extends the utility of tension infiltrometer data to a large class of hydraulic conductivity models and provides an in situ technique to determine suitable parameters values for these models.

Introduction

The introduction of hazardous products at a large number of waste sites in the United States has motivated investigators to develop vadose zone models of water flow and solute transport to predict the fate of contaminants. Central to the application of these models is the determination of the soil unsaturated hydraulic conductivity-water pressure relationship. The objective of this paper is to develop an in situ field technique that uses tension infiltrometer data to determine parameters of the Gardner and the Brooks and Corey hydraulic conductivity-water pressure models.

As shown in Figure 1, the major components of a tension infiltrometer are a bubbling tube that controls the water supply pressure at the soil surface, a water reservoir which empties as water flows into the soil, and a disc that contains a porous membrane to establish hydraulic continuity with the soil. For a prescribed water supply pressure, the infiltration rate is determined by reading the falling water level in the reservoir. The infiltrometer shown in Figure 1 does not require pushing a ring into the soil to produce one-dimensional flow. Driving a ring disturbs the

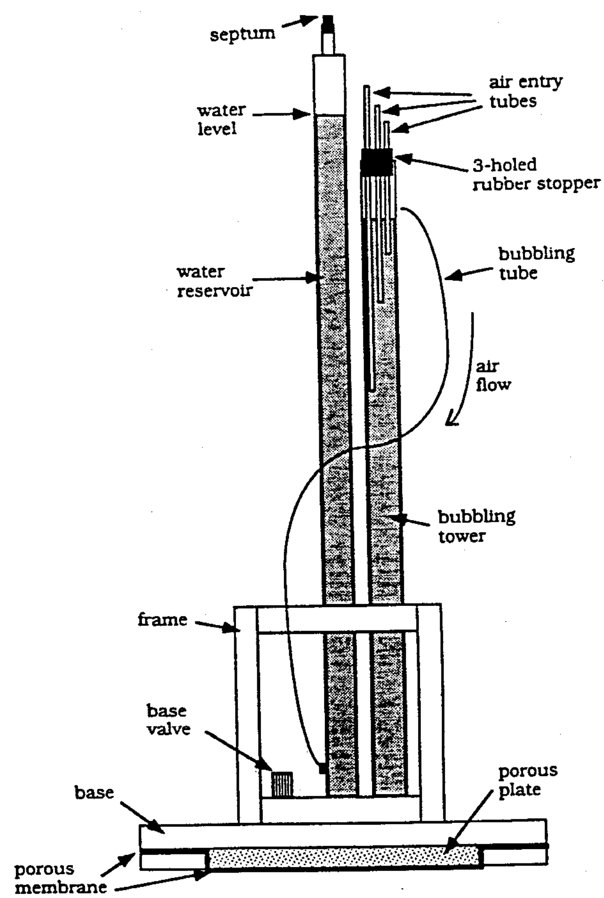

Fig. 1. Schematic diagram of the tension infiltrometer.

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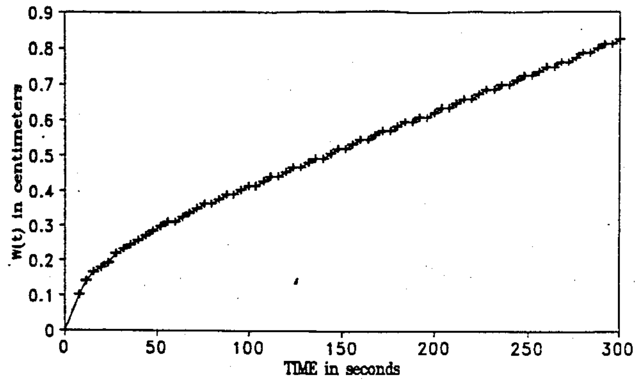

Fig. 2. Cumulative infiltration versus time for the first infiltration test.

soil and may alter its pore structure. Hence, the infiltration underneath the infiltrometer disc is three-dimensional, transient at early times, and nonlinear, because of the dependence of hydraulic conductivity on soil-water pressure.

To determine the hydraulic conductivity-water pressure relationship and related soil hydraulic parameters, soil physicists have investigated several methods to analyze the data collected from a tension infiltrometer. The usefulness of these methods varies depending on their assumptions and field practicality. Also, depending on the type of field observations involved, methods of analysis include steady-state methods, transient methods, and hybrid methods. Because sorptivity measurements require monitoring the soil moisture content, Ankeny et al. (1989) disregarded the data on the transient stage of infiltration. Their methodology to determine the hydraulic conductivity depends on measuring the quasi-steady infiltration rate, and uses Wooding's equation (Wooding, 1968) to model the three-dimensional infiltration beneath the infiltrometer disc. The derivation of Wooding's equation relies on an exponential relationship between soil-water pressure (ψ) and conductivity [$K(\psi)$]. In this relationship, the influence of water pressure on hydraulic conductivity is assumed to be adequately represented by a parameter called the sorptive number (α). Reynolds and Elrick (1991) suggested a modification to the equation of Wooding because, for some soils, α is not a constant over the entire range of water supply pressures [i.e., $\alpha = \alpha(\psi)$]. Again, their approach relies solely on analyzing the quasi-steady stage of the infiltration process to develop the hydraulic conductivity-water pressure relationship. White and Perroux (1989) designed a methodology that requires sorptivity measurements and employs the data on the transient infiltration. This method requires measurements of sorptivity at two levels of water supply pressure for a uniform initial moisture content. To insure uniform initial moisture content, cores of soil should be removed from the field and returned to air-dry between measurements. This procedure increases the experimental time significantly.

In this paper, an in situ hybrid method is developed to determine the parameters of the Gardner, and the Brooks and Corey models of $K(\psi)$. In contrast to the available methods described above, data on both the transient and quasi-steady-state infiltration are utilized. The determination of the parameter values for the Brooks and Corey model of $K(\psi)$ requires a single evaluation of sorptivity using the data collected during one infiltration test. The sorptivity provides additional information on the relationship between capillarity and hydraulic conductivity. The method described here, which employs transient and steady-state data simultaneously, should lend more credibility to the $K(\psi)$ relationship developed with the tension infiltrometer. It also permits the calibration of a greater variety of $K(\psi)$ models.

Description of the data collected during an infiltration test and operation of the tension infiltrometer are presented in section 1. Use of these data to determine $K(\psi)$ models is demonstrated in section 2.

1. Data Collection and Description of Infiltration Processes

Data Collection and Operation of the Infiltrometer

All infiltrometers operate on the same physical principles. The infiltrometer shown in Figure 1 is supplied by Soil Measurements Systems. The negative water supply pressure at the soil surface is controlled by the air entry tubes in the bubbling tower. Each of the air entry tubes is calibrated to generate a different level of negative water supply pressure. In practice, in addition to the ponded condition at zero pressure, negative water supply pressures of 3, 6, and 15 cm have proven convenient and useful across a variety of soil conditions. The falling water level in the reservoir, which is monitored with a tensicorder, provides a temporal description of the cumulative water recharge [$W(t)$] entering the soil for a fixed water supply pressure (ψ_0) at the soil surface. Figure 2 illustrates the behavior of $W(t)$ for a single infiltration test. This infiltration test is conducted four times for the calibrated levels of water supply pressure. These data are then analyzed to determine soil hydraulic conductivity.

Description of Infiltration Processes

The unconfined flow under the infiltrometer disc is three-dimensional. Water moves by capillarity and gravity in the vertical direction and by capillarity in the horizontal direction. Also, the infiltration is transient at early times because of the deficit between the applied water pressure (or water content) at the surface and the initial soil conditions. Philip (1969) recognized two distinct time scales to describe the infiltration. The first time scale accounts for the interaction between capillarity and the flow geometry, whereas the second time scale accounts for the interaction between capillarity and gravity. At early times, Philip (1966, 1969) suggested that infiltration is not distinguishable from a one-dimensional absorption problem, and the cumulative infiltration [$W(t)$] should satisfy the relationship:

$$\lim_{t \to 0} W(t) = S\,t^{1/2} \qquad (1)$$

where $S = S(\theta_0, \theta_n)$ is the sorptivity between the applied water content (θ_0) and the initial water content (θ_n), and t is time. The change in soil moisture content ($\Delta\theta = \theta_0 - \theta_n$) can be monitored in the field with a Time Domain Reflectrometry probe. The slope of W(t) versus $t^{1/2}$ is the sorptivity as demonstrated in Figure 3.

At later times, a quasi-steady infiltration develops in the soil under the infiltrometer disc (Wooding, 1968). The infiltration rate (q_s) is the slope of the plot of W(t) versus t upon reaching a quasi-steady state (see Figure 2). For three-dimensional, unconfined steady flow from a circular ring of radius r, Wooding (1968) provided the following approximate expression for the infiltration rate (q_s):

$$q_s(\psi_0) = K(\psi_0) + K(\psi_0)\frac{4}{\pi r \alpha} \qquad (2)$$

where $q_s(\psi_0)$ and $K(\psi_0)$ are the infiltration rate and the hydraulic conductivity at the applied water supply pressure (ψ_0), r is the radius of the infiltrometer disc, and α is the sorptive number (Philip, 1969, 1985; Wooding, 1968). Physically, the sorptive number is a relative measure of the importance of gravity and capillarity forces during infiltration. Coarse soils, where gravity tends to dominate, have large values of α; whereas fine-textured soils, where capillarity is important, have small values of α. Elrick et al. (1989) suggested that α varies between 0.04 cm^{-1} for unstructured fine-textured soils and 0.36 cm^{-1} for coarse and gravelly sands. The influence of the geometry of the infiltrometer disc on infiltration is reflected through the second term on the right-hand side of equation (2). When the radius of the infiltrometer is very large, this term tends to zero and the flow becomes essentially one-dimensional, driven by gravity.

Equations (1) and (2) are introduced to simplify the description of infiltration processes under the infiltrometer disc. In the following section, these relationships are used to determine unsaturated hydraulic conductivity models.

2. Determination of Soil Hydraulic Conductivity Models

Soil scientists and hydrologists have suggested several hydraulic conductivity models [$K(\psi)$]. For a particular soil, the choice of $K(\psi)$ should depend on how well this model fits field observations.

Case of the Gardner Exponential Model of $K(\psi)$

To linearlize the three-dimensional flow equation and achieve the closed-form solution in equation (2), Wooding (1968) used Gardner's exponential model of hydraulic conductivity:

$$K(\psi) = K_s \exp(\alpha\psi) \qquad (3)$$

where K_s is the saturated hydraulic conductivity. The goodness-of-fit of Gardner's exponential model to field data is controversial. Philip (1969) suggested that equation (3) "does model in a reasonably convincing way the quite generally observed rapid and nonlinear decrease of hydraulic conductivity with water pressure." Yeh et al. (1985) and Montoglou and Gelhar (1987) adopted this exponential model to develop the stochastic theory of unsaturated flow in heterogeneous soils. Also, Warrick (1974) and Raats

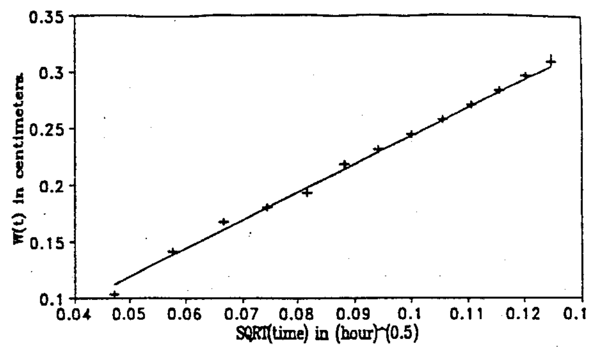

Fig. 3. Soil sorptivity is determined from the early time behavior of infiltration.

(1976) used the Gardner model in deterministic studies of steady and transient multidimensional flow.

To determine K_s and α, the parameters of Gardner's exponential model, equation (3) is substituted into equation (2) and the natural logarithm is applied on both sides of the resulting equation to obtain:

$$\ln q_s(\psi_0) = \alpha\psi_0 + \ln\left[K_s\left(1 + \frac{4}{\pi r \alpha}\right)\right] \qquad (4)$$

The slope of the linear regression of $\ln(q_s)$ versus ψ_0 is α, the sorptive number. The saturated hydraulic conductivity (K_s) is determined using the relationship:

$$K_s = \frac{q_s(0)}{1 + (4/\pi r \alpha)} \qquad (5)$$

The method described above is implemented on data from tension infiltrometer tests conducted at five locations. The soil is sandy loam (62% sand, 21% silt, and 17% clay). The calibrated parameter values for Gardner's model, K_s and α, are presented in Table 1. The calibrated parameter values of α and K_s encountered at this site are typical of undisturbed sandy loam soil (Elrick et al., 1989).

Case of the Brooks and Corey Model of $K(\psi)$

Another widely used $K(\psi)$ model is the one suggested by Brooks and Corey (1964). Good agreement between laboratory measurements and this model was found by Brooks and Corey (1964) and Laliberte et al. (1966). The Brooks and Corey model is favored among many groundwater hydrologists and is adopted in existing numerical codes of water flow and solute transport in the unsaturated zone (e.g., USGS, 1987; EPA, 1989).

Table 1. Parameters of the Gardner's Exponential Relationship

Test number	K_s *($cm.hr^{-1}$)*	α *(cm^{-1})*	S *($cm.hr^{-1/2}$)*
1	16.3	0.26	4.13
2	15.4	0.22	2.85
3	13.4	0.24	2.07
4	17.2	0.21	2.40
5	16.2	0.25	2.43

In the Brooks and Corey model, $K(\psi)$ is given as:

$$K(\psi) = K_s(\psi_d/\psi)^{2+3\lambda} \qquad \psi < \psi_d \qquad (6)$$
$$K(\psi) = K_s \qquad \psi \geq \psi_d$$

where ψ_d is the entry pressure, and λ is a pore-size distribution index. Brooks and Corey found that for typical porous media, λ is about 2. Soils with well-developed structure have values of λ less than 2, and homogeneous sands normally have values of λ greater than 2. To calibrate the Brooks and Corey model using the tension infiltrometer data, expressions that relate ψ_d and λ to the determined S and α [see equations (1) and (2)] are required.

Philip (1985) proposed the expression:

Hc : $$\alpha^{-1} = (1/K_s) \int_{-\infty}^{0} K(\psi)\, d\psi \qquad (7)$$

to relate the sorptive number (α) to any $K(\psi)$ model because it ensures that quasi-linear solutions derived using equation (3) provide good estimates of soil hydraulic properties. He showed mathematically that equation (7) is the optimal solution for α. The substitution of equation (6) into (7) and integration yields:

Hc : $$\alpha^{-1} = \frac{(3\lambda + 2)}{(3\lambda + 1)} (-\psi_d) \qquad (8)$$

Equation (8) has two unknowns, so another relationship that involves the measured sorptivity is needed to reach a solution.

Relationship Between Sorptivity and Parameters of the Brooks and Corey Model

To relate sorptivity to soil hydraulic conductivity, Philip (1969) suggested using D_*, a mean weighted diffusion coefficient, that results from matching the observed (one-dimensional) absorption rate with the absorption rate determined with the constant D_*. This matching yields the solution for D_*:

$$D_* = \left(\frac{\pi^{1/2} S}{2\Delta\theta}\right)^2 \qquad (9)$$

Crank (1975) revealed that D_* controls the absorption rate if it assumes the form:

$$D_* = (5/3)(\Delta\theta)^{-5/3} \int_{\theta_0}^{\theta_n} (\theta - \theta_n)^{2/3} D(\theta)\, d\theta \qquad (10)$$

He showed that the relationship in equation (10) holds to within an accuracy of 1% for nonlinear diffusivity [$D(\theta)$] functions. For the case of the Brooks and Corey model, the diffusivity function is given as:

$$D(\theta) = \left[\frac{(-\psi_d) K_s}{\lambda \Delta\theta}\right] \left(\frac{\theta - \theta_n}{\Delta\theta}\right)^{[(2\lambda+1)/\lambda]} \qquad (11)$$

The substitution of equation (11) into (10) yields:

$$D_* = \frac{-5\psi_d K_s}{(11\lambda + 3)\Delta\theta} \qquad (12)$$

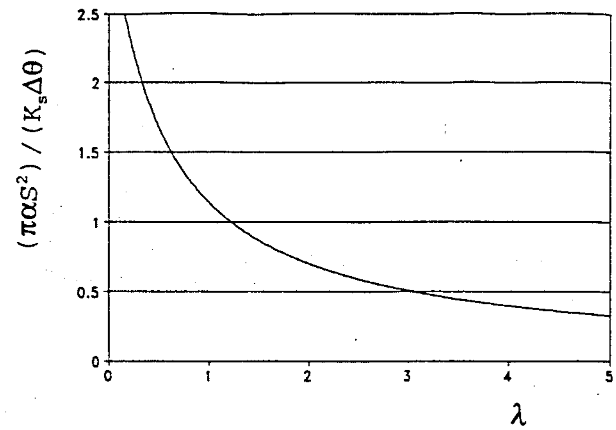

Fig. 4. Calibration curve for λ.

Table 2. Parameters of the Brooks and Corey's Conductivity Relationship

Test number	K_s *(cm.hr⁻¹)*	$-\psi_d$ *(cm)*	λ
1	16.3	3.35	1.96
2	15.4	3.78	1.32
3	13.4	3.64	1.96
4	17.2	4.26	2.53
5	16.2	3.30	1.22

Using equation (9) in (12) provides the algebraic relationship:

$$\frac{\pi S^2}{20\Delta\theta K_s} = \frac{-\psi_d}{11\lambda + 3} \qquad (13)$$

Equations (8) and (13) can be solved simultaneously for the two unknowns ψ_d and λ. Substituting (8) in (13) yields:

$$\frac{20(3\lambda + 1)}{(11\lambda + 3)(3\lambda + 2)} = \frac{\pi S^2 \alpha}{K_s \Delta\theta} \qquad (14)$$

A plot of ($\pi\alpha S^2/K_s\Delta\theta$) versus λ is shown in Figure 4. To estimate parameter values of the Brooks and Corey model, λ is first determined using the graph in Figure 4. The substitution of λ into equation (8) then provides ψ_d.

The calibration procedures formulated here were implemented on the tension infiltrometer data. The estimated parameter values are documented in Table 2. The pore-size distribution index is within the range suggested by Brooks and Corey (1964) for field soils.

Conclusion

A simple field method is developed to determine soil unsaturated hydraulic conductivity using tension infiltrometer data. Applications of this method were demonstrated for two cases of unsaturated hydraulic conductivity relationships. Simultaneous use of the transient and steady-state measurements of infiltration provide a physically based, in situ alternative to determine the parameter values for $K(\psi)$ models. The influence of water pressure on hydraulic conductivity is determined using two measured parameters, the sorptivity, calculated from the early transient behavior of infiltration, and the sorptive number, determined from the later quasi-steady infiltration rate. This approach lends more credibility to K(ψ) relationships developed with the tension infiltrometer. The method described here provides a practical field approach for the determination of K(ψ) relationships needed for vadose zone models of water flow and solute transport.

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